Mathematical Modelling of Carbon Fee and Dividend (aka Climate Income)

Dave Waltham, October 2nd 2021

The CCL-Europe climate income calculator assumes that household emissions increase with household income. The simplest possible mathematical model that achieves this is

$$E_0 = aI \tag{1}$$

where E_0 is household emissions (at the model start) and *I* is household disposable income. I determine *a* by assuming that an adult on median income has the average per capita emissions. This gives a different value for *a* for each European country.

In addition, emissions change with time, *t*, by a factor *R* so that emissions are

$$E(t) = R(t)E_0 \tag{2}$$

Carbon-fees for each person are then

$$F = PE \tag{3}$$

where *F* is the fee and *P* is the price. The dividend (i.e. climate income per adult) is found by assuming that an individual with average emissions will receive a dividend equal to their carbon-fees. Hence, the dividend per person is

$$D = PR\overline{E_0} \tag{4}$$

where $\overline{E_0}$ is the average emissions per person at the beginning. The calculator assumes that children receive half this dividend.

Emission reductions are assumed to be driven by the carbon price (i.e. *R* decreases as *P* increases). Many choices could be made for the mathematical form of this but I have used the sigmoidal function

$$R = [\ 0.5 + 0.5\cos\left(\frac{\pi P}{P_{max}}\right)\ ] \qquad P<P_{max} \tag{5}$$

$$= 0 \qquad P \geq P_{max}$$

where P_{max} is the price needed to eliminate emissions entirely. This sigmoidal form fits the REMI model of the US economy for P_{max}~$400/tonne CO_2. The CCL-Europe calculator uses a slightly more conservative estimate that P_{max}=€400/tonne CO_2.

Finally, the model needs a price policy and I assume a simple ramping up with time

$$P = P_{start} + \alpha t \tag{6}$$

where P_{start} is the initial price, α is the price increase each year and *t* is time since carbon pricing was introduced. More specifically, I have used P_{start}=€15/tonne and α=€10/tonne/year.