Mathematical Modelling of Carbon Fee and Dividend (aka Climate Income)

Dave Waltham, October 2nd 2021

The CCL-Europe climate income calculator assumes that household emissions increase with household income. The simplest possible mathematical model that achieves this is

\[ E_0 = aI \]  

(1)

where \( E_0 \) is household emissions (at the model start) and \( I \) is household disposable income. I determine \( a \) by assuming that an adult on median income has the average per capita emissions. This gives a different value for \( a \) for each European country.

In addition, emissions change with time, \( t \), by a factor \( R \) so that emissions are

\[ E(t) = R(t)E_0 \]  

(2)

Carbon-fees for each person are then

\[ F = PE \]  

(3)

where \( F \) is the fee and \( P \) is the price. The dividend (i.e. climate income per adult) is found by assuming that an individual with average emissions will receive a dividend equal to their carbon-fees. Hence, the dividend per person is

\[ D = P\bar{E}_0 \]  

(4)

where \( \bar{E}_0 \) is the average emissions per person at the beginning. The calculator assumes that children receive half this dividend.

Emission reductions are assumed to be driven by the carbon price (i.e. \( R \) decreases as \( P \) increases). Many choices could be made for the mathematical form of this but I have used the sigmoidal function

\[ R = \begin{cases} 0.5 + 0.5 \cos \left( \frac{\pi P}{P_{\text{max}}} \right) & P < P_{\text{max}} \\ 0 & P \geq P_{\text{max}} \end{cases} \]  

(5)

where \( P_{\text{max}} \) is the price needed to eliminate emissions entirely. This sigmoidal form fits the REMI model of the US economy for \( P_{\text{max}} \sim \$400/\text{tonne CO}_2 \). The CCL-Europe calculator uses a slightly more conservative estimate that \( P_{\text{max}} = €400/\text{tonne CO}_2 \).

Finally, the model needs a price policy and I assume a simple ramping up with time

\[ P = P_{\text{start}} + \alpha t \]  

(6)

where \( P_{\text{start}} \) is the initial price, \( \alpha \) is the price increase each year and \( t \) is time since carbon pricing was introduced. More specifically, I have used \( P_{\text{start}} = €15/\text{tonne} \) and \( \alpha = €10/\text{tonne/} \text{year} \).